

Investigating Generalized Parton Distribution in Gravity Dual

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Abstract

Generalized parton distribution (GPD) contains rich information of partons in a hadron, including transverse profile, and is also non-perturbative information necessary in describing a variety of hard processes, such as meson leptonproduction and double deeply virtual Compton scattering (DDVCS). In order to unveil non-perturbative aspects of GPD, we study DDVCS at small x in gravitational dual description. Using the complex spin j -plane representation of DDVCS amplitude, we show that GPD is well-defined and can be extracted from the amplitude even in the strong coupling regime. It also turns out that the saddle point value in the j -plane representation plays an important role; there are two phases in the imaginary part of amplitude of DDVCS and GPD, depending on relative position of the saddle point and the leading pole in the j -plane, and crossover between them is induced by the change of the kinematical variables. The saddle point value also directly controls kinematical variable dependence of many observables in one of the two phases, and indeed the dependence is qualitatively in nice agreement with HERA measurements. Such observation that the gravity dual shares basic properties of the real world QCD suggests that information from BFKL theory might be used to reduce error in the gravity dual predictions of the form factor and of GPD. This article also serves as a brief summery of a preprint arXiv:1105.2999.

1 Introduction

AdS/CFT correspondence and its extension to non-conformal theories have been exploited for study of non-perturbative aspects of strongly coupled gauge theories. Hadron spectra, coupling constants among them and chiral symmetry breaking have been studied intensively in the literature by using gravitational dual descriptions with smooth infra-red non-conformal geometries. The gravitational dual approach can be used, however, to study not just *static* properties of strongly coupled gauge theories, but also *scattering* of hadrons. Indeed, string theory or dual resonance model was originally constructed to describe scattering of hadrons. Qualitative aspects of hadron scattering can be obtained in gravitational dual descriptions, if the background geometry (target space) of string theory is chosen properly [1, 2, 3].

In this article, we will study 2-body to 2-body scattering of a hadron and a virtual photon at high energy in gravitational dual descriptions. This process is called double deeply virtual Compton scattering (DDVCS). When the final state photon is on-shell, it is called deeply virtual Compton scattering (DVCS), and is accessible in experiments [4]. Because of QCD factorization theorem [5], the DVCS or DDVCS amplitude is obtained as a convolution of generalized parton distribution (GPD) [6] and a hard kernel, the latter of which can be calculated in perturbative QCD. GPD itself (at a certain factorization scale), however, is a non-perturbative object in nature, and cannot be calculated in perturbative QCD. Even in determining it by using experimental data, its profile needs to be parametrized¹ based on proper understanding on non-perturbative dynamics behind confinement. We thus use gravitational dual descriptions to extract theoretical understanding on the GPD profile.

It is not that we just use a well-developed technique to calculate a specific scattering amplitude (or GPD) in this article, however. This article clarifies structure of Pomeron “exchange” amplitudes, how to organize them, as well as their field-theory interpretation. We find that a saddle point value of the scattering amplitude in complex spin j -plane representation is a key concept in organizing Pomeron amplitudes and in understanding kinematical variable dependence of the scattering amplitude. Based on this understanding, sharp cross-over behavior is expected in the photon-hadron 2-to-2 scattering amplitude in small x limit.

This article is meant to be a brief summary of reference [8]. To keep this letter short enough, we extracted material mainly from §5 of [8], and only minimum from other sections, imagining people in perturbative QCD community as primary readers of this letter. More theoretical aspects of the scattering amplitude in gravity dual, as well as more detailed account of the materials in this letter, are found in [8].

¹See [7] for review articles, which also have extensive list of literatures.

2 Amplitude in Gravity Dual

In order to calculate hadron–virtual photon scattering amplitude in gravitational dual, one needs to adopt a certain holographic model. Since the real world QCD turns from weak coupling at high energy into strong coupling at infrared, it is desirable to have a holographic model that is faithful to string theory where AdS curvature becomes larger than string scale toward UV boundary. Such a model becomes even more realistic, if spontaneous chiral symmetry breaking is implemented in it. Our primary goal in this article, however, is not in pursuing precision in numerical calculation (as lattice QCD does) by setting up a perfectly realistic gravitational dual description. An appropriate set-up that suits the best for one’s purpose should depend on the purpose.

We will focus on qualitative aspects of hadron–virtual photon scattering amplitude at small x (at high center-of-mass energy). Since small x physics is dominated by gluon, not by quarks and anti-quarks, we do not find it a crucial element to implement flavor in the gravitational set up for the purpose of this article. For explicit calculation, we adopt the hard wall model [2], which is type IIB string theory on $W \times \text{AdS}_5$ for some 5-dimensional manifold W with AdS_5 cut off at finite radius at infrared. Such a crude treatment of infrared geometry is sufficient for our qualitative study [2], and the choice of W becomes irrelevant (at least directly) for sufficiently small x [8]. Since it is almost straightforward to see how the AdS_5 curvature and running of dilaton expectation value affects various observables in explicit calculations based on the hard wall model, one can also learn what happens in gravitational dual models that are asymptotically conformal or asymptotically free without carrying out calculations separately on these models.

As an analogy of the electromagnetic global $U(1)$ symmetry of QCD, we take a global symmetry of W in the gravitational dual. Since we are interested in the Compton tensor² of QCD,

$$i(2\pi)^4 \delta^4(p_2 + q_2 - p_1 - q_1) T^{\mu\nu} = - \int \int d^4x d^4y e^{-iq_2 \cdot x} e^{+iq_1 \cdot y} \langle h(p_2) | T \{ J^\mu(x) J^\nu(y) \} | h(p_1) \rangle, \quad (1)$$

we use the bulk-to-boundary propagator of an AdS_5 vector field associated with a Killing vector of W in calculating the matrix element involving the global symmetry current. As for the target hadron in the gravity dual, we use a Kaluza–Klein state of a dilaton, whose wavefunction is given by a Bessel function in the hard wall model. Thus, the leading order contribution in $1/N_c$ expansion is given by a closed string sphere amplitude with four NS–NS

²In our convention, $\eta_{\mu\nu} = \text{diag}(-, +, +, +)$. Let us remark that the Compton tensor $T^{\mu\nu}$ in this letter is defined differently from one in [8]; the Lorentz indices μ, ν are interchanged.

string vertex operator insertions [2].³

As we consider cases where the initial state “photon” or both the initial and final state “photons” are highly virtual, that is, $q_1^2 \gg \Lambda^2$ or $q_1^2, q_2^2 \gg \Lambda^2$, the “photon”–hadron scattering amplitude $T^{\mu\nu}$ can be decomposed into various contributions through operator product expansion of $J^\mu(x)$ and $J^\nu(y)$ in QCD language. Such a decomposition still holds true in strongly coupled gauge theories (and hence in gravitational dual), except that the anomalous dimensions of operators in the expansion may be quite different from what one expects in the weak coupling regime. Reference [2] noted that the operators that are twist-2 in the weakly coupled regime still appear in the operator product expansion even in the strongly coupled regime, and their contributions to the Compton tensor $T^{\mu\nu}$ dominate at sufficiently small x ; this is because the “twist-2” contribution corresponds to exchange of leading Regge trajectory containing graviton in gravity dual language [9, 3]. We will thus focus on small x hadron–virtual photon scattering in gravity dual to study non-perturbative behavior of the “twist-2” contribution.

Before writing down the Pomeron contribution to the scattering amplitude explicitly, let us note that the Compton tensor is described by five structure functions $V_{1,2,\dots,5}$ as in [10],

$$T^{\mu\nu} = V_1 P[q_2]^{\mu\rho} P[q_1]_\rho^\nu + V_2 (p \cdot P[q_2])^\mu (p \cdot P[q_1])^\nu + V_3 (q_1 \cdot P[q_2])^\mu (q_2 \cdot P[q_1])^\nu + V_4 (q_1 \cdot P[q_2])^\mu (p \cdot P[q_1])^\nu + V_5 (p \cdot P[q_2])^\mu (q_2 \cdot P[q_1])^\nu - A \epsilon^{\mu\nu\rho\sigma} q_{1\rho} q_{2\sigma}, \quad (2)$$

for a scalar target hadron, because of gauge invariance. In parity-preserving theory, $A = 0$. In the limit of purely forward scattering, the two structure functions of deep inelastic scattering are restored from $\text{Im } V_1(x, \eta, t, q^2) \rightarrow F_1(x, q^2)$ and $(q^2/(2x)) \times \text{Im } V_2(x, \eta, t, q^2) \rightarrow F_2(x, q^2)$. Here, we introduced a convenient notation

$$P[q]_{\mu\nu} = \left[\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right]. \quad (3)$$

In this article, we will use the following notations,

$$q^\mu = \frac{(q_1 + q_2)^\mu}{2}, \quad p^\mu = \frac{(p_1 + q_2)^\mu}{2}, \quad x = -\frac{q^2}{2p \cdot q}, \quad \eta = -\frac{q \cdot (q_1 - q_2)}{2p \cdot q}, \quad (4)$$

and $t = -(q_1 - q_2)^2$ and $s = W^2 = -(q + p)^2$.

³ The target hadron which is dual to a Kaluza–Klein state of a dilaton is a glueball. The case of a meson target can also be studied in the same way if we use open strings. For the case of a baryon target, we should use D -brane in the gravity dual. We will see that the saddle point value and singularities in the complex j -plane representation are important in describing the amplitude. Because they do not depend on the target hadron wavefunctions, they are expected to be unchanged even if the species of target hadron is replaced.

In the generalized Bjorken limit, $\Lambda^2, |t| \ll q_1^2$, and for x much smaller than unity, the Pomeron contribution to the five structure functions are given by I_0 and I_1 [8] as in

$$\begin{aligned} V_1 &\simeq \frac{1}{2}I_1, & V_2 &\simeq \frac{2x^2}{q^2}(I_0 + I_1), & V_3 &\simeq \frac{x^2}{2q^2}(I_0 + I_1), \\ V_4 &\simeq \frac{x}{q^2}I_1, & V_5 &\simeq \frac{x}{q^2}I_1; \end{aligned} \quad (5)$$

I_0 and I_1 are given for vanishing skewedness η in the form of

$$I_i(x, \eta, t, q^2) \simeq \frac{c'_s}{2\kappa_5^2} \frac{\pi}{2R^3} \int dz \sqrt{-g(z)} \int dz' \sqrt{-g(z')} P_{\gamma^* \gamma^*}^{(i)}(z) \mathcal{K}(s, t, z, z') P_{hh}(z'). \quad (6)$$

For vanishing skewedness, the Pomeron kernel \mathcal{K} is [3]⁴

$$\begin{aligned} \mathcal{K}(s, t; z, z') &\simeq -4R\sqrt{\lambda} \int_{-\infty}^{\infty} d\nu \frac{1}{2\pi i} \int_{C_1(\nu)} dj \frac{1 + e^{-i\pi j}}{\sin \pi j} \frac{1}{\Gamma^2(j/2)} \\ &\quad \left(\frac{\alpha' \tilde{s}}{4} \right)^j \frac{1}{j - j_\nu} e^{-jA(z)} \Psi_{i\nu}^{(j)}(t, z) e^{-jA(z')} \Psi_{i\nu}^{(j)}(t, z'); \end{aligned} \quad (7)$$

the integration contour in the complex j -plane encircles the pole $j = j_\nu$, and once the residue of this pole is picked up, a relation

$$j = j_\nu \equiv 2 - \frac{4 + \nu^2}{2\sqrt{\lambda}} \quad (8)$$

sets the (analytically continued) relation between spin j and anomalous dimension $\gamma = i\nu - j$ of “twist-2” operators in the large 't Hooft coupling $\lambda \gg 1$ regime [3]. $e^{2A(z)} = (R/z)^2$ is the warp factor in the AdS₅ part of the metric in the hard wall model,

$$ds^2|_{\text{AdS}_5} = e^{2A(z)} (\eta_{\mu\nu} dx^\mu dx^\nu + (dz)^2), \quad (9)$$

and $\sqrt{-g}$ in (6) is that of this metric of 5-dimensional spacetime. R is the AdS radius, and the infrared cut off of the hard wall model at $z = 1/\Lambda$ sets the confinement scale Λ . $\tilde{s} = e^{-A(z)} e^{-A(z')} s$, and α' is the slope parameter of the Type IIB string theory. $\Psi_{i\nu}^{(j)}(t, z)$ in (7) is the Pomeron wavefunction in the spin j channel, which is given by

$$\Psi_{i\nu}^{(j)}(t, z) = ie^{A(j-2)} \sqrt{\frac{\nu}{2R \sinh \pi \nu}} \left[\sqrt{\frac{I_{-i\nu}(\sqrt{-t}/\Lambda)}{I_{i\nu}(\sqrt{-t}/\Lambda)}} I_{i\nu}(\sqrt{-t}z) - \sqrt{\frac{I_{i\nu}(\sqrt{-t}/\Lambda)}{I_{-i\nu}(\sqrt{-t}/\Lambda)}} I_{-i\nu}(\sqrt{-t}z) \right] \quad (10)$$

⁴ More careful discussion on the choice of integration contour is given in [11, 8]. A pedagogical explanation of the origin of $1/\Gamma^2(j/2)$ factor is also given in [8].

in the hard wall model.⁵

The impact factor $P_{hh}(z')$ of the target hadron side is given by the normalizable mode wavefunction of the target hadron, as in $P_{hh}(z') = c_\phi(\Phi(z'))^2$. On the virtual “photon” side, the bulk-to-boundary propagator (non-normalizable wavefunction) of the graviton associated with the Killing vector of W is used; in the hard wall model, they are

$$P_{\gamma^*\gamma^*}^{(1)}(z) = c_J^2 R^2 e^{-2A(z)} [(q_1 z)(K_1(q_1 z))][(q_2 z)K_1(q_2 z)], \quad (11)$$

$$P_{\gamma^*\gamma^*}^{(0)}(z) = \frac{c_J^2 R^2 e^{-2A}}{q^2} [(q_1^2 z)(K_0(q_1 z))][(q_2^2 z)K_0(q_2 z)] \quad (12)$$

for I_1 and I_0 , respectively. κ_5^2 is a constant of a theory of mass dimension -3 and is proportional to N_c^2 . c'_s , c_ϕ and c_J are dimensionless constants of order unity. See [8] for their definitions.

3 Structure and Behavior of the Amplitude

3.1 Complex j -plane amplitude, Pomeron vertex and form factor

Before discussing kinematical parameter (x, t, q^2) dependence of the DDVCS amplitude in gravity dual, let us clarify a couple of conceptual issues associated with Pomerons. Using the explicit form of the Pomeron kernel (7) and Pomeron wavefunctions (10), amplitudes I_i ($i = 0, 1$) in (6) can be rewritten (see [8] for details) as

$$I_i(x, \eta = 0, t, q^2) \simeq \sqrt{\lambda} \int_{-\infty}^{\infty} d\nu \left[-\frac{1 + e^{-\pi i j_\nu}}{\sin \pi j_\nu} \right] \frac{1}{\Gamma^2(j_\nu/2)} [C^{(i)}(j, q)]_\mu [A_{hh}]_\mu, \quad (13)$$

where

$$[C^{(i)}(j, q)]_\mu = \left[\frac{1}{R^3} \int dz \sqrt{-g(z)} P_{\gamma^*\gamma^*}^{(i)}(z) e^{-2A(z)} \left(\frac{z}{R} \right)^{i\nu} (Rz)^{j_\nu} \right] \times (R\mu)^{i\nu - j_\nu}, \quad (14)$$

$$[A_{hh}]_\mu \simeq \frac{1}{(R\mu)^{i\nu - j_\nu}} \times \left[\frac{c'_s}{\kappa_5^2} \int dz' \sqrt{-g(z')} P_{hh}(z') \left[\frac{e^{-2A(z')} W^2}{4\sqrt{\lambda}} \right]^{j_\nu} \right. \\ \left. \left[\frac{e^{(j_\nu - 2)A(z')}}{K_{i\nu}(\sqrt{-t}R)} \left(K_{i\nu}(\sqrt{-t}z') - \frac{K_{i\nu}(\sqrt{-t}/\Lambda)}{I_{i\nu}(\sqrt{-t}/\Lambda)} I_{i\nu}(\sqrt{-t}z') \right) \right] \right]; \quad (15)$$

a parameter μ of mass dimension $+1$ is introduced in (14, 15) in a way the observables I_i are unaffected. One can change the integration variable of (13) from ν to $j = j_\nu$; now the amplitudes I_i are given by integration over the complex j -plane, and the contour becomes the one in Figure 1 (a).

⁵Dirichlet boundary condition was imposed at the infrared boundary $z = 1/\Lambda$, just to make expressions simpler.

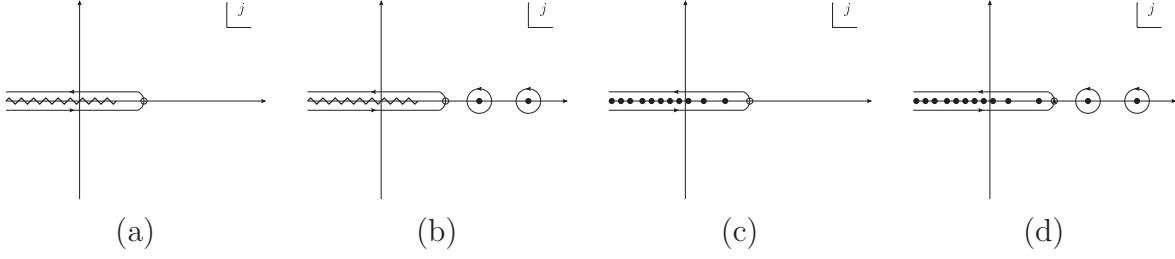


Figure 1: Singularities and integration contours in the complex j -plane. The hard-wall model is assumed for (a) negative t and (b) sufficiently large positive t , while a holographic model for asymptotic free running coupling is assumed in (c) with a smaller t , and in (d) with a larger t . Black dots are poles, wiggling lines in (a, b) are branch cuts, and open circles in (a–d) denote saddle points of the amplitude on the complex j -plane.

The factor $[C^{(i)}(j, q)]_\mu$ is now regarded as a function of j , and also depends on q^2 and μ , but not on t or x . Its asymptotic form for $q^2 \gg \Lambda^2$ is given by

$$[C^{(i)}(j, q)]_\mu \simeq c_J^2 \left(\frac{\mu}{q}\right)^{\gamma(j)} \frac{1}{(q^2)^j} \bar{c}_{i\nu_j}^{(i)}, \quad (16)$$

with a dimensionless constant of order unity $\bar{c}_{i\nu_j}^{(i)}$ that depends only on j . Here, $\gamma(j) \equiv i\nu_j - j$, and $\nu_j = \nu(j)$ is the inverse function of $j = j_\nu$ (8). x dependence and t dependence of the amplitudes I_i come from the other factor $[A_{hh}]_\mu$. It can be rewritten as

$$[A_{hh}]_\mu \simeq c'_s \left(\frac{W^2}{4\sqrt{\lambda}}\right)^j \left(\frac{\Lambda}{\mu}\right)^{\gamma(j)} g_{i\nu_j}^h(\sqrt{-t}/\Lambda) \simeq c'_s \left(\frac{1}{4\sqrt{\lambda}x}\right)^j (q^2)^j \left(\frac{\Lambda}{\mu}\right)^{\gamma(j)} g_{i\nu_j}^h(\sqrt{-t}/\Lambda), \quad (17)$$

where $g_{i\nu_j}^h(\sqrt{-t}/\Lambda)$ is a dimensionless function of j and $(\sqrt{-t}/\Lambda)$. For the final expression, we used $W^2 \simeq q^2/x$ which holds at small x . Combining both, one finds that

$$I_i \simeq c'_s \sqrt{\lambda} \int_{-\infty-i\epsilon}^{-\infty+i\epsilon} dj \frac{|\partial \nu_j / \partial j| c_J^2}{\Gamma^2(j/2)} \left[-\frac{1 + e^{-\pi i j}}{\sin(\pi j)}\right] \left(\frac{1}{4\sqrt{\lambda}x}\right)^j \left(\frac{\Lambda}{q}\right)^{\gamma(j)} \bar{c}_{i\nu_j}^{(i)} g_{i\nu_j}^h(\sqrt{-t}/\Lambda). \quad (18)$$

This is in the form of inverse Mellin transformation, and the integration variable j is identified with the complex angular momentum (complex spin).⁶

Now, physical meaning of the separation between $[C_{i\nu_j}^{(i)}]_\mu$ and $[A_{hh}]_\mu$ (or $g_{i\nu_j}^h(\sqrt{-t}/\Lambda)$) is clear. By changing the integration contour in the j -plane, (13, 18) can be rewritten as

$$I_i \simeq \sum_{j \in 2\mathbb{N}} \frac{4\sqrt{\lambda} \left|\frac{\partial \nu_j}{\partial j}\right|}{\Gamma^2(j/2)} \left[c_J^2 \left(\frac{\mu}{q}\right)^{\gamma(j)} \frac{1}{(q^2)^j} \bar{c}_{i\nu_j}^{(i)} \right] c'_s \left(\frac{2q \cdot p}{4\sqrt{\lambda}}\right)^j \left[\left(\frac{\Lambda}{\mu}\right)^{\gamma(j)} g_{i\nu_j}^h(\sqrt{-t}/\Lambda) \right]. \quad (19)$$

⁶Since we restrict ourselves to the scattering at $\eta = 0$, total derivative operators in field-theory language do not contribute to the OPE of the scattering amplitude. Thus, there is no subtleties in what this j is here.

This is regarded as an OPE form of I_i . The first factor in $[\cdots]$, which comes from $[C^{(i)}(j, q)]_\mu$, is regarded as the Wilson coefficient of OPE for a spin $j \in 2\mathbb{N}$ operator; the parameter μ is now identified with the renormalization scale, because of its appropriate scaling behavior determined by the anomalous dimension $\gamma(j)$ of the “twist-2” spin j operator. The second factor in $[\cdots]$ is identified with the spin j form factor, which is the coefficient of the $[p^{\mu_1} \cdots p^{\mu_j}]$ term of the hadron matrix element of the spin j operator renormalized at the scale μ . The gravity dual expression (15) justifies such an interpretation [12].

Knowing physical meaning of these factors in the *scattering amplitude* (13) in a gravity dual model, one can define a *GPD* even in the model, which corresponds to a strongly coupled gauge theory. GPD as a function of x and t (we only consider the $\eta = 0$ case in this letter) is defined as an inverse Mellin transform of form factors of twist-2 spin j operators (the second factor in $[\cdots]$ of (19)). The scattering amplitude I_i is given by convolution of this GPD, inverse Mellin transform of the Wilson coefficient and that of the signature factor $-[1 + e^{-\pi i j}]/\sin(\pi j)$, just like in perturbative QCD factorization formula. The inverse Mellin transform of the signature factor gives rise to a light-cone singularity of a propagating parton (like the one in [13]), even in the gravity dual description. The GPD determined in this way is essentially⁷ the same as $\text{Im } I_i$, with q^2 of $\text{Im } I_i$ replaced by the renormalization scale μ^2 ; thus, various statements on $\text{Im } I_i$ in the rest of this section are also applied to the GPD after q^2 is replaced by μ^2 .

Now that the field theory OPE interpretation of the gravity dual amplitude (13) is clarified, let us go back to the amplitude (13) and explicit expressions (14, 15) once again. We will now clarify how this string theory amplitude on a warped background is related to the traditional Regge phenomenology ansatz. It should be noted that the DDVCS amplitudes I_i in gravity dual (18) do not have a Pomeron pole like $1/(j - \alpha(t))$ in their j -plane representation apparently. There was once a pole $1/(j - j_\nu)$ at the stage of (7), but it is gone in (18), after picking the residue to evaluate an integral in (7). Nevertheless, one can see that the expression (18) may have, in fact, many poles in the j -plane, rather than a single pole or none.

To see this, we can use Kneser–Sommerfeld expansion of Bessel functions in the hard wall

⁷Since GPD is defined as the inverse Mellin transform of form factors of “twist-2” spin j operators, it would become different when the normalization of the operators were changed in a j -dependent manner. We do not pay such a careful attention in this article. We claim similarity between $\text{Im } I_i$ and GPD after replacement of q^2 by μ^2 only at this level of precision.

model to rewrite $[\Gamma_{hh\mathbb{P}^*}(j, t)]_\mu \equiv (\Lambda/\mu)^{\gamma(j)} g_{i\nu_j}^h(\sqrt{-t}/\Lambda)$ as [8]

$$[\Gamma_{hh\mathbb{P}^*}(j, t)]_\mu = \sum_{n=1}^{\infty} \frac{-2}{t - m_{j,n}^2} \frac{\gamma_{hh\mathbb{P}n}(j)}{\Lambda^{j-2}} \frac{\left(\frac{m_{j,n}}{2}\right)^j \left(\frac{m_{j,n}}{2\mu}\right)^{\gamma(j)}}{[J'_{i\nu_j}(j_{i\nu_j,n})]} \frac{2}{\Gamma(i\nu_j)} \left[\frac{R^3}{\kappa_5^2}\right]^{1/2}, \quad (20)$$

$$\gamma_{hh\mathbb{P}n}(j) = \frac{1}{\kappa_5^2} \int dz \sqrt{-g} P_{hh}(z) e^{-2jA} \psi_n^{(j)}(z) \times (R\Lambda)^j. \quad (21)$$

The Pomeron trajectory (that contains graviton) of the Type IIB string theory on 10-dimensions (or on AdS_5 after dimensional reduction on W) gives rise to a Kaluza–Klein tower of infinitely many Pomeron trajectories in hadron scattering on 3+1 dimensions. These trajectories are labeled by the Kaluza–Klein excitation level n ; the masses of spin $j \in 2\mathbb{N}$ hadrons are $m_{j,n}$, and their wavefunctions on AdS_5 are $\psi_n^{(j)}(z)$. The factor $1/(t - m_{j,n}^2)$ in (20) becomes a t dependent pole in the j -plane, the Pomeron pole, for any one of n 's. In the hard wall model, the Pomeron trajectory ($j = \alpha_{\mathbb{P},n}(t)$) relation set by $t = m_{j,n}^2$ and the Pomeron wavefunction for the n -th trajectory are obtained holomorphically in j (not just for $j \in 2\mathbb{N}$) as in

$$m_{j,n} = \Lambda j_{i\nu_j,n}, \quad \psi_n^{(j)}(z) = e^{(j-2)A} \frac{J_{i\nu_j}(m_{j,n}z)}{J'_{i\nu_j}(j_{i\nu_j,n})} \left[\frac{\kappa_5^2}{R^3}\right]^{1/2}, \quad (22)$$

where, $j_{\mu,n}$ is the n -th zero of Bessel function $J_\mu(z)$. $m_{j,n}$ is read out from the denominator in (15); the wavefunction $\psi_n^{(j)}(z)$ satisfies an equation of motion of a spin j field on AdS_5 , just like (10) does. Although explicit expressions above rely heavily on the hard wall model, conceptual understanding itself is quite general, and is applicable at least to any asymptotically conformal gravity dual models.

Therefore, gravity dual descriptions of strongly coupled gauge theories come up with a following picture of Pomeron exchange amplitude. Individual Pomerons in the Kaluza–Klein tower couple to the target hadron with a coupling $\gamma_{hh\mathbb{P}n}(j)$ in (21), which do not show any power-law fall-off behavior in large negative t . Only after all the Pomeron couplings $\gamma_{hh\mathbb{P}n}(j)$ and Pomeron propagators $1/(t - m_{j,n}^2) \propto 1/(j - \alpha_{\mathbb{P},n}(t))$ are combined as in (20), do we obtain what we might call a “Pomeron form factor” $[\Gamma(j, t)]_\mu$, which has a power-law behavior in $\sqrt{-t}$ (see (26)). Such a relation between a form factor of a conserved current and a combination of a Kaluza–Klein tower of hadrons, three point couplings and decay constants has been known for fixed spins (such as $j = 1$ and $j = 2$) [14]. The relation (20) is regarded as an analytic continuation in j of the one for graviton (spin $j = 2$).

3.2 Saddle point in the j -plane

It was a conventional wisdom of traditional Regge phenomenology that behavior of hadron scattering amplitudes at high energy are governed by the position of singularities in the complex j -plane. The same is true in gravity dual description of strongly coupled gauge theories. Singularities in scattering amplitudes in the complex j -plane representation depend on choice of gravity dual models. In case of the hadron–virtual “photon” scattering, however, the scattering amplitude can be approximated at saddle point in the j -plane (within a certain kinematical region which we call “saddle point phase” in §3.3). In this case, the expression of the amplitude becomes not directly dependent on the singularities, and hence, detail of gravitational dual is irrelevant. In this subsection, we employ the hard wall model, and study the behavior of this scattering amplitude.

In the hard wall model, there are no isolated poles in the complex j -plane for negative t —physical kinematics—except the branch cut that extends to negative j along the real axis, Figure 1(a); $(t - m_{j,n}^2)$ never vanishes for $t < 0$. Thus, the j integral of (18) along the contour in Figure 1 (a) is evaluated by the saddle point method for small x [11]. For $0 \leq -t \lesssim \Lambda^2$, the saddle point value of j is given by⁸

$$j^* = j_{\nu^*}, \quad i\nu^* = \frac{\sqrt{\lambda} \ln(q/\Lambda)}{\ln \left[(q/\Lambda)/(\sqrt{\lambda}x) \right]}, \quad (23)$$

and

$$\text{Im } I_i(x, \eta = 0, t, q^2) \sim \left(\frac{1}{\sqrt{\lambda}x} \right)^{j^*} \left(\frac{\Lambda}{q} \right)^{\gamma(j^*)} g_{i\nu_{j^*}}^h(\sqrt{-t}/\Lambda); \quad (24)$$

the form factor $g_{i\nu}^h(\sqrt{-t}/\Lambda)$ has a dimensionless non-zero limit of order unity when $-t \rightarrow 0$; it begins to fall off in power-law⁹ as

$$g_{i\nu}^h(\sqrt{-t}/\Lambda) \simeq \left(\frac{\Lambda}{\sqrt{-t}} \right)^{-\gamma(j)+2\Delta-2} \tilde{g}_{i\nu_j}^h \quad (26)$$

for larger momentum transfer $\Lambda^2 \ll -t$. Here, Δ is the scaling dimension of the scalar field on AdS_5 containing the target hadron h , and $\tilde{g}_{i\nu}^h$ is a t -independent (but ν_j -dependent) constant

⁸ The integrand of the Pomeron kernel (7) is reliable at $|j| \sim \mathcal{O}(1)$, but not at $|j| \gtrsim \sqrt{\lambda}$ [9]. Therefore, we note that kinematical variables (x , q^2 and t) are required to be consistent with $j^* \sim \mathcal{O}(1)$.

⁹Thus, for $\Lambda^2 \ll -t$, the saddle point becomes

$$i\nu^*(q/\Lambda, x, -t \gg \Lambda^2) = \sqrt{\lambda} \frac{\ln(q/\sqrt{-t})}{\ln \left(\frac{q/\sqrt{-t}}{\sqrt{\lambda}x} \right)}. \quad (25)$$

of order unity. Note, in particular, that the t -dependence of the scattering amplitude is given by the form factor that is once analytically continued to complex j -plane and then evaluated at the saddle point. The Regge factor $(W^2)^j$ of string theory amplitude justifies focusing on a small range of j (or ν) around the saddle point value at high-energy scattering; the power-law behavior in $\sqrt{-t}$ follows from the power-law wavefunction of the target hadron $P_{hh}(z')$ and exponential cut-off of the Pomeron wavefunction, $K_{i\nu}(\sqrt{-t}z)$ in (15) in particular, in the limited range of ν_j .

The saddle point method provides a good approximation to the scattering amplitude for $\ln(1/x)/\sqrt{\lambda} \gg 1$. It should be noted, however, that it allows us to keep all-order contributions in $i\nu^* = \sqrt{\lambda} \ln(q/\Lambda)/\ln(1/x)$, which is not necessarily small and can be as large as $\mathcal{O}(\lambda^{1/4})$. Thus, amplitudes and observables are expressed as functions of $i\nu^*$ (or j^*). This makes easy to understand their dependence of kinematical variables (x, t, q^2) .

Equation (24) clearly shows the importance of the value of the saddle point of the j -plane amplitude. To see this more explicitly, let us define

$$\gamma_{\text{eff}}(x, t, q^2) = \frac{\partial \ln[x I_i(x, \eta = 0, t, q^2)]}{\partial \ln(\Lambda/q)}, \quad \lambda_{\text{eff}}(x, t, q^2) = \frac{\partial \ln[x I_i(x, \eta = 0, t, q^2)]}{\partial \ln(1/x)}. \quad (27)$$

It is straightforward to see that they are given by

$$\gamma_{\text{eff}}(x, t, q^2) = \gamma(j^*), \quad \lambda_{\text{eff}}(x, t, q^2) = j^* - 1. \quad (28)$$

These effective exponents γ_{eff} and λ_{eff} depend on kinematical variables x, q^2 and t only through the saddle point value j^* . The ratio $\rho = \text{Re } I_i / \text{Im } I_i = \tan(\frac{\pi}{2}(j^* - 1))$ is also related directly to the saddle point value j^* .

We can see from (23, 25) that j^* becomes large for large q^2 and small for small x . Thus, at a given renormalization scale μ^2 (replace q^2 in $\text{Im } I_i$), GPD in gravity dual still increases in the DGLAP evolution (that is, $\gamma_{\text{eff}} < 0$) for small enough x such that the saddle point value j^* is still less than 2. Even at such a small value of x , however, GPD eventually begins to decrease (that is, γ_{eff} becomes positive) for large enough μ^2 . Such a behavior of GPD—qualitatively the same as in the real world QCD—in the DGLAP evolution was anticipated in [2]; this is indeed realized for finite μ^2 in gravity dual, when both q^2 (μ^2) and x dependence are included in the saddle point approximation. The other parameter λ_{eff} characterizing the x evolution is known to increase gradually for larger q^2 in the real world QCD [15]. As already seen in [16], it does follow from gravity dual as well; we understand that this phenomenon is also essentially due to the increase of the saddle point value j^* for larger q^2 . The same behavior is also obtained in perturbative QCD (See [17]).

The t dependence of the scattering amplitude is characterized by the slope parameter of the forward peak (also known as t -slope parameter), which we define for non-skewed

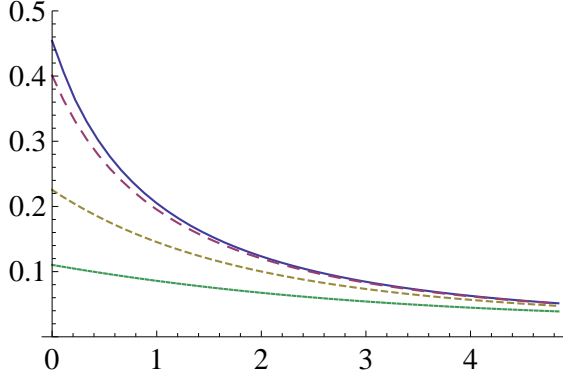


Figure 2: (color online) Slope B of the forward peak in DDVCS. The dimensionless value $B \times \Lambda^2$ is shown as a function of $i\nu^*$ (23); from top to bottom, blue (solid line) curve is for $\sqrt{-t}/\Lambda \simeq 0.01\text{--}0.1$, red (long dashed) one for $\sqrt{-t}/\Lambda = 1.$, yellow (dashed) one for $\sqrt{-t}/\Lambda = 3.$, and green (short dashed) one for $\sqrt{-t}/\Lambda = 6.$ For more details, see [8].

scattering as

$$B_i(x, \eta = 0, t, q^2) = 2 \frac{\partial}{\partial t} \ln \text{Im } I_i(x, \eta = 0, t, q^2). \quad (29)$$

The t -dependence (and hence the slope parameter) comes entirely from the form factor for the physical kinematical region $t \leq 0$ in the hard wall model. The t -slope parameter at $t = 0$ in such a case can be regarded as the charge radius square of the hadron under “spin- j^* probe”. Explicit expressions for the form factor in the hard wall model allow us to calculate the t -slope parameter; see Figure 2. The larger the spin j^* (and hence $i\nu^*$), the smaller the slope. Therefore, through (23), the slope parameter decreases for larger q^2 , a prediction of a gravity dual model which cannot be made within perturbative QCD.

We can also see from (23, 25) that the saddle point value j^* depends weakly on $\ln(1/x)$ or $\ln(W^2/\Lambda^2)$ than on $\ln(q^2/\Lambda^2)$ for small x , and the dependence is in the opposite direction. Thus, the $\ln(1/x)$ dependence (or $\ln(W^2/\Lambda^2)$ dependence) of the slope parameter B must be weaker than its $\ln(q^2/\Lambda^2)$ dependence. This property of B , shared by γ_{eff} , λ_{eff} and ρ , is an immediate consequence of the fact that the scattering amplitude is well approximated by the saddle point method on the j -plane integral. This is a fairly robust feature of the saddle point approximation, and does not rely on specific details of the hard wall model.

The saddle point turns out to be an important concept also in the scattering amplitude $\text{Im } I_i$ in the impact parameter space, which is obtained by taking a Fourier transform in the transverse direction of the momentum transfer $(p_1 - p_2)$. The x -dependent parton density profile in the transverse direction obtained in this way [18] in gravity dual shows Gaussian

profile at large impact parameter b , but is larger than the simple Gaussian form for smaller b ([8]; see also [16]; deviation from the simple Gaussian profile is an immediate consequence of the fact that the 4D leading trajectory $j = \alpha_{\mathbb{P},n=1}(t)$ is not perfectly linear). This core of larger parton density has approximately a linear exponential profile, $e^{-m_{j^*,1}b}$. The effective mass scale $m_{j^*,n=1} = m_{n=1}^{(\nu^*)}$ gradually changes as a function of b , and the linear exponential form smoothly turns into the Gaussian form for larger b , when $i\nu^*$ becomes of order unity. See [8] for more.

3.3 Pole–Saddle Point Crossover

Although we saw that the saddle point method well approximated the DDVCS amplitude for physical kinematical region $t \leq 0$ in the hard-wall model, it does not in general. Even in small x , whether or not the scattering amplitude is well approximated by the saddle point method, depends on singularities of the amplitude in the j -plane representation, and hence on the gravity dual model one considers, and also the values of the kinematical variables x, q^2 and t . Although all the asymptotically conformal gravity dual models have a branch cut that is stretched to large negative j , there may also be some isolated poles in the j -plane as in Figure 1 (b). The hard wall model does not have such a pole for physical kinematical region $t \leq 0$ (there are for sufficiently positive t), but there may be some for other UV conformal models that have different (and faithful to string theory construction) infrared geometry. Even more interesting are gravity dual models that are asymptotically free, where the cut is replaced by isolated singularities (Figure 1 (c, d)) [3].

When the saddle point (open circle in the figure) has a larger real part than any one of the singularities in the complex j -plane, then the integration contour in the j -plane should simply be chosen so that it passes through the saddle point, as in Figure 1 (a, c). When some of the singularities have larger real parts than the saddle point value j^* , however, it is more convenient to take the contour as in Figure 1 (b, d), so that the scattering amplitude is given by contributions from finite number of isolated Pomeron poles $j = \alpha_n(t)$ ($n = 1, 2, \dots$) and by a continuous integration over a contour passing through the saddle point. We refer to the two situations as saddle point phase and leading pole phase (or leading singularity phase), respectively. Such observables as λ_{eff} , γ_{eff} , ρ and B exhibit totally different dependence on the kinematical variables x, q^2 and t in the two phases. In a given theory (i.e., in a given gravity dual model), one always enters into the saddle point phase for sufficiently large q or sufficiently negative t . In asymptotically free theories, it is likely that the leading singularity phase also exists for sufficiently small x and not so large negative t , even in the physical kinematical region $t \leq 0$.

The transition between the two phases is not singular but is a (smooth) crossover for finite x . This is because the saddle point approximation is never exact, and the “saddle point” should be thought of as a sort of diffuse object for finite x . Subleading singularities may also give rise to significant corrections to the amplitude simply given by the leading pole $j = \alpha_1(t)$ for finite x , too. The transition becomes a singular phase transition only in the extreme small x limit.

4 Lessons to Learn

It is true that gravity dual calculation employs a background that corresponds to large 't Hooft coupling even at energy scale much larger than the hadronic scale Λ . Still, there are surprisingly many qualitative features in the gravity dual hadron–virtual “photon” scattering amplitude that are in common with the scattering amplitude in the real world QCD. Scattering amplitudes in gravity dual have $\ln(q/\Lambda)$ and $\ln(1/x)$ scaling governed by $\gamma_{\text{eff}} = \gamma(j^*)$ and $\lambda_{\text{eff}} = j^* - 1$ in the saddle point phase, and this is the same qualitatively as the prediction of the saddle point method in perturbative QCD, as we have already seen in §3.2. The only difference between gravity dual and real world QCD is in the choice of anomalous dimension, $\gamma(j)$. Qualitative features are shared by both, and are controlled by the saddle point value j^* .

Qualitative features in t -dependence also show agreements. The gravity dual amplitude continues to the power-law fall-off behavior at large momentum transfer $\Lambda^2 \ll -t$. This property, which is expected to hold in the real world QCD theoretically [19] and confirmed experimentally, was difficult to be consistent with the traditional Regge phenomenology, but this problem is now overcome in gravity dual on *warped* spacetime (cf. [1, 2]). Moreover, the t -slope parameter of (29) and its result in Figure 2 for $\eta = 0$ in gravity dual at saddle point phase nicely agrees with that in DVCS differential cross section [20], in that the slope parameter B decreases for larger $\ln(q/\Lambda)$, and is less sensitive to $\ln(1/x)$ or $\ln(W/\Lambda)$. Such observation suggests the (analytically continued) spin j form factors $[\Gamma_{hh\mathbb{P}^*}(j, t)]_\mu$ in both a gravity dual model and the real QCD are similar to each other.

With so many basic qualitative features that gravity dual shares with the real world QCD, it is thus tempting to try to extract some lessons from the hadron–virtual “photon” amplitude in gravity dual. The origin of such similarity at the qualitative level becomes clear in the complex j -plane representation, where GPD is given by inverse Mellin transformation:

$$H(x, \eta = 0, t; \mu^2) \sim \int \frac{dj}{2\pi i} \left(\frac{1}{x}\right)^j \left[\left(\frac{\Lambda}{\mu}\right)^{\gamma(j)} g_{i\nu_j}^h(\sqrt{-t}/\Lambda) \right]. \quad (30)$$

Indeed, it is always possible to describe scattering amplitude by the j -plane integral in any theories, independent of whether the scattering is based on the real QCD or on the strongly coupled gauge theory studied in gravity dual; this is because Mellin transformation is only a mathematical transformation. This j -plane integral also comes from OPE, notion of which is well-defined even in strongly coupled theories [2]. GPD in the j -plane representation (30) is given by dropping the Wilson coefficient of OPE from the scattering amplitude $\text{Im } I_i$ (13), so the spin j form factor (reduced matrix element of twist-2 spin j operator), which is the content of $[\dots]$ in (30), determines GPD. The spin j form factor is decomposed into two parts: RG evolution part $(\Lambda/\mu)^{\gamma(j)}$, and form factor at renormalization scale $\mu = \Lambda$, $g_{i\nu_j}^h(\sqrt{-t}/\Lambda)$; both show common properties in the real QCD and in gravity dual. The anomalous dimensions of the twist-2 spin j operator $\gamma(j)$ in both theories are qualitatively similar [3], and $g_{i\nu_j}^h(\sqrt{-t}/\Lambda)$ has the power-law fall-off behavior at $-t \gg \Lambda^2$ in common.

The behavior of GPD is determined by the saddle point, or alternatively, by the leading singularity depending on which phase a set of parameters (x, t, μ^2) sits in. This classification is applicable in any theories, not just in gravity dual. Then it is important to know which phase a given set (x, t, μ^2) sits in. As we have pointed out, the behaviors of γ_{eff} and λ_{eff} observed in HERA for DIS [15] are successfully explained by the predictions of the saddle point phase and are inconsistent with the prediction of the leading pole phase (or the leading singularity phase) [8]. Therefore, it is very likely that the (most of) kinematical region of DVCS that has been explored in HERA measurements is in the saddle point phase,¹⁰ and GPD is approximately given by

$$H(x, \eta = 0, t, \mu^2) \sim \left(\frac{1}{x}\right)^{j^*} \left(\frac{\Lambda}{\mu}\right)^{\gamma(j^*)} g_{i\nu_{j^*}}^h(\sqrt{-t}/\Lambda). \quad (31)$$

Most of the observed properties of the t -slope parameter B of DVCS in HERA [20] can be understood only from the fact that the kinematical region is in the saddle point phase (see [8]). A GPD model with a specific choice of $g_{i\nu_{j^*}}^h(\sqrt{-t}/\Lambda)$ in [22] belongs to this category.¹¹

One can also see that the saddle point expression (31) automatically satisfies a requirement

¹⁰ In the standard parametrization of DVCS cross section $\frac{d\sigma_{\text{DVCS}}(\gamma^* p \rightarrow \gamma p)}{dt} \sim \frac{\alpha_{QED}^2}{\Lambda^4} \times \left(\frac{W}{\Lambda}\right)^\delta \left(\frac{\Lambda^2}{q^2}\right)^n$, the parameters (δ, n) are given by $\delta = 4(j^* - 1)$ and $n = \gamma(j^*) + 2j^*$ in the saddle point phase. Thus, the saddle point phase implies rise of δ for larger q^2 . HERA measurement [21] gives $\delta = 0.44 \pm 0.19$ for $q^2 = 2.4 \text{ GeV}^2$, $\delta = 0.52 \pm 0.09$ for $q^2 = 3.2 \text{ GeV}^2$, $\delta = 0.75 \pm 0.17$ for $q^2 = 6.2 \text{ GeV}^2$, $\delta = 0.84 \pm 0.18$ for $q^2 = 9.9 \text{ GeV}^2$, and $\delta = 0.76 \pm 0.22$ for $q^2 = 18 \text{ GeV}^2$ in ZEUS, and $\delta = 0.61 \pm 0.10 \pm 0.15$ for $q^2 = 8 \text{ GeV}^2$, $\delta = 0.61 \pm 0.13 \pm 0.15$ for $q^2 = 15.5 \text{ GeV}^2$, and $\delta = 0.90 \pm 0.36 \pm 0.27$ for $q^2 = 25 \text{ GeV}^2$, in H1.

¹¹Reference [22] introduces an ansatz $g_{i\nu_j}^h \sim (j - \alpha(t))^{-1} (1 - t/\Lambda^2)^{-p}$, inspired by a leading Pomeron pole $(j - \alpha(t))^{-1}$ and a power-law fall-off for $(-t) \gg \Lambda^2$. Our result (20) is conceptually different from this model; each Pomeron pole term with a Kaluza-Klein excitation level n does not show the behavior of power-law fall-off, but the power-law (26) appears only after summing all the Kaluza-Klein tower of Pomeron pole terms.

that GPD should be consistent with DGLAP evolution, because μ -evolution is correctly taken into account in the j -plane expression (30). This is a nontrivial requirement on GPD modeling in general. One can consider, for example, a GPD profile given by PDF (GPD at $t = 0$) multiplied by some form factor at a given renormalization scale [23]:

$$\left[\left(\frac{1}{x} \right)^{j^*} \left(\frac{\Lambda}{\mu} \right)^{\gamma(j^*)} \right] \times \frac{1}{(1 - B(x)t)^p}, \quad (32)$$

where $B(x) = \alpha'(1-x)^3 \ln(1/x) + \dots$, and α' and p are parameters. The profile of GPD like this are not stable under DGLAP evolution. On the other hand, the GPD under the saddle point approximation (31) is given by the PDF multiplied by a spin j form factor evaluated at the saddle point value $j = j^*$. The saddle point value j^* depends on x and the factorization/renormalization scale μ . This result obviously takes into account renormalization effects, and hence is stable/reliable at any renormalization scale.

The remaining task is to determine the spin j form factor at renormalized point $\mu = \Lambda$, $g_{iv_j}^h(\sqrt{-t}/\Lambda)$ as a holomorphic function of j . This is along the line of the collinear factorization approach (dual parametrization) to the modeling of GPD[24]. Derivation of $g_{iv_j}^h(\sqrt{-t}/\Lambda)$ from the first principle is an impossible task in perturbative QCD, because of the non-perturbative origin of the form factor, and this is also hard in lattice simulation, because there is practically no way of finding analytic continuation of integer spin matrix elements into complex j . An alternative is to use predictions from the gauge/string duality, and a crude way is to use the prediction of the hard wall model derived in §3.1 as it is. Indeed, as we saw, the hard wall model can explain decreasing slope parameter B of DVCS for large q^2 , observed in HERA [20]. It is also possible to use more realistic gravity dual models for similar calculation, where at least we might want to require the model to have asymptotic free running for certain energy range (as in [25]) still with large 't Hooft coupling.

If one wants to consider a gravity dual model that is truly dual to the real world QCD (if there is any), then it should run into a problem in its UV region of the geometry because of large curvature. This problem of gravitational description, however, may be alleviated by borrowing the understanding of perturbative QCD. Such strategy may not be totally nonsense. We saw that the singularities of the form factor in the j -plane are important in determining GPD, and gravity dual with asymptotic free running suggests that the singularities are infinitely many poles¹² [3]. The BFKL theory in perturbative QCD with a running coupling effect also suggests infinitely many poles in the j -plane [26]. Now, let us examine

¹² These poles correspond to trajectories of Kaluza-Klein modes in radial direction (z) of a single graviton trajectory in 10 dimensions (or on AdS_5). On top of this tower structure, there is yet another tower structure of trajectories associated with the daughter trajectories of stringy excitations on 10 dimensions.

how sensitive the position of the poles predicted from gravity dual are to the unreliable large curvature geometry in the UV region. In gravitational descriptions, each Pomeron pole has its wavefunction on the holographic coordinate, and the Pomeron wavefunction becomes localized more and more into the IR region of the holographic radius when $\text{Re } j$ of the pole increases. Therefore, the poles in large $\text{Re } j$ are determined mainly by IR physics, and position of poles predicted by gravity dual should be reliable, while the poles in small $\text{Re } j$ are quite sensitive to the unreliable geometry in the UV region. As for such smaller $\text{Re } j$ poles, however, the position of the poles predicted by the BFKL theory (with asymptotically free running) will be reliable. Thus, by using both predictions from the gravity dual and the BFKL theory, the poles in the j -plane may be properly determined.

In order to determine GPD completely, not only the position of the poles but also complete profile of the spin j form factor are required. The spin j form factor is given by integrating Pomeron wavefunction and impact factor in gravity dual, and in fact, also in the BFKL theory; the integration is carried out over the holographic radius z in gravity dual, whereas it is done over gluon transverse momentum k_\perp in the BFKL theory. The similar structure in the k_\perp factorization formula and the gravity dual scattering amplitude (6) has been pointed out, and identification of gluon transverse momentum k_\perp in the BFKL theory with holographic radius z^{-1} in the gravity dual is suggested [27, 3]. Thus, one can retain the integration over the holographic radius in gravity dual in the IR (large z) region. The integration in the UV region may be replaced by that over k_\perp coordinate in the BFKL theory; this large k_\perp region is where perturbative QCD is reliable.

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